II. Filter Fundamentals

• Purpose
  – Frequency selection
    • Low pass, high pass, band pass, band stop, notch, etc.

• Applications
  – Many applications in communications, instrumentation, signal processing
Low Pass and High Pass Filter

Single Time Constant (STC) Networks

- Attenuation beyond 3dB frequency not steep
Ideal Filter Frequency Response

- LP
- HP
- BP
- BS

Ref. Sedra and Smith, Figure 12.2
Specifications of Non-ideal LP

- $A_{\text{max}} = \text{maximum allowable attenuation in passband}$
- $A_{\text{min}} = \text{minimum required attenuation in stopband}$
- $\omega_p = \text{passband edge}$
- $\omega_s = \text{stopband edge}$

Ref. Sedra and Smith, Figure 12.3
Specifications of Non-ideal BP

- $A_{\text{max}}$ = maximum allowable attenuation in passband
- $A_{\text{min}}$ = minimum required attenuation in stopband
- $\omega_{p1}, \omega_{p2}$ = passband edges
- $\omega_{s1}, \omega_{s2}$ = stopband edge

Ref. Sedra and Smith, Figure 12.4
General Filter Transfer Function

- Transfer function in s-domain
  \[ T(s) = \frac{V_o(s)}{V_i(s)} \]

- Transfer function for physical frequencies
  \[ T(j\omega) = |T(j\omega)|e^{j\phi(\omega)} \]

- Polynomial expansion of \( T(s) \)
  - \( N=1 \) filter order
  - \( N \geq M \)
  \[ T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \cdots + a_0}{s^N + b_{N-1} s^{N-1} + \cdots + b_0} \]

  \[ T(s) = \frac{a_M (s - z_1) \cdot (s - z_2) \cdots (s - z_M)}{(s - p_1) \cdot (s - p_2) \cdots (s - p_M)} \]
General Filter Transfer Function

- Factor numerator and denominator

\[ T(s) = \frac{a_M(s - z_1) \cdot (s - z_2) \cdot \cdots \cdot (s - z_M)}{(s - p_1) \cdot (s - p_2) \cdot \cdots \cdot (s - p_M)} \]

- Numerator roots: \( z_1, z_2, \ldots, z_M \)
  - Transfer function zeros (transmission zeros)
  - Also, N-M transmission zeros at \( s = \infty \)

- Denominator roots: \( p_1, p_2, \ldots, p_N \)
  - Transfer function poles (natural modes)
Characteristics of Trans. Fct. Poles, Zeros

- May be real or complex
- Complex zeros and poles occur in conjugate pairs
- Transmission zeros usually purely imaginary at stopband frequencies
- Poles have negative real part for stable filters
- Represent on ‘pole-zero’ plot
Pole-Zero Pattern

- Poles and zeros plotted on complex s-plane
- All poles complex conjugate except for odd order filter (one pole on real axis)

\[ N = \]  
\[ M = \]  
\[ N - M = \text{zeros at } s = \infty \]
Some Filter Types

Butterworth (maximally flat magnitude)

This filter has the flattest possible pass-band magnitude response. Attenuation is −3 dB at the design cutoff frequency. Attenuation above the cutoff frequency is a moderately steep 20-dB per decade per pole. The pulse response of the Butterworth filter has moderate overshoot and ringing.

Chebyshev (equal ripple magnitude)

Note: Mr. Chebyshev's name is also transliterated Tschebychev, Tschebyscheff or Tchebyssheff. This filter type has steeper attenuation above the cutoff frequency than Butterworth. This advantage comes at the penalty of amplitude variation (ripple) in the passband. Unlike Butterworth and Bessel responses, which have 3-dB attenuation at the cutoff frequency, Chebyshev cutoff frequency is defined as the frequency at which the response falls below the ripple band. For even-order filters, all ripple is above the 0-dB-gain dc response, so cutoff is at 0-dB (see Figure 1.) For odd-order filters, all ripple is below the 0-dB-gain dc response, so cutoff is at −(ripple) dB (see Figure 2.) For a given number of poles, a steeper cutoff can be achieved by allowing more pass-band ripple. The Chebyshev has even more ringing in its pulse response than the Butterworth.

Bessel (maximally flat time delay)

(Also called Thomson.) Due to its linear phase response, this filter has excellent pulse response (minimal overshoot and ringing). For a given number of poles, its magnitude response is not as flat, nor is its attenuation beyond the −3-dB cutoff frequency as steep as the Butterworth. Although it takes a higher-order Bessel filter to give a magnitude response which approaches that of a given Butterworth filter, the pulse response fidelity of the Bessel filter may make the added complexity (because of additional filter sections) worthwhile.

Butterworth LP Filter

- **Characteristics**
  - Monotonically decreasing transmission
  - All transmission zeros at \( \omega = \infty \) (all pole)
  - Very flat response near \( \omega = 0 \)

- **Transfer function**

\[
|T(j\omega)| = \frac{1}{\sqrt{1 + \varepsilon^2 \left( \frac{\omega}{\omega_p} \right)^{2N}}}
\]

Ref. Sedra and Smith, Figure 12.9
Butterworth LP Filter

- At $\omega = \omega_p$
  \[ A_{\text{max}} = A(\omega = \omega_p) = -20 \log \frac{1}{\sqrt{1 + \varepsilon^2}} \]

- At $\omega = \omega_s$
  \[ A_{\text{min}} = A(\omega = \omega_s) = -20 \log \sqrt{1 + \varepsilon^2 \left( \frac{\omega_s}{\omega_p} \right)^{2N}} \]

\[ \varepsilon = \sqrt{10 \left( \frac{A_{\text{max}}}{10} - 1 \right)} \]
Butterworth LP Filter

- Transfer function

\[ T(s) = \frac{K\omega_0^N}{(s - p_1)(s - p_2) \cdots (s - p_N)} \]

where \( \omega_0 = \omega_p \left( \frac{1}{\varepsilon} \right) \)

\[ |T(\omega)| \]

<table>
<thead>
<tr>
<th>( \omega_p )</th>
<th>( \omega_s )</th>
</tr>
</thead>
</table>

Diagram showing the frequency response and poles of the filter.
Chebyshev LP Filter

- Characteristics: ‘Equi-ripple response in passband
  - Monotonically decreasing transmission in passband
  - All transmission zeros at $\omega = \infty$

$$|T(j\omega)| = \begin{cases} \frac{1}{\sqrt{1 + \epsilon^2 \cos^2 \left[ N \cos^{-1} \left( \frac{\omega}{\omega_p} \right) \right]}, & \omega \leq \omega_p \end{cases}, \omega \geq \omega_p$$

Ref. Sedra and Smith, Figure 12.12
Chebyshev LP Filter

- At $\omega = 0$ \quad $A(\omega = 0) = -20 \log 1 = 0 dB$, for $N=\text{odd}$

\[= -20 \log \frac{1}{\sqrt{1 + \varepsilon^2}}, \text{ for } N=\text{even}\]

- At $\omega = \omega_p$ \quad $A_{\text{max}} = A(\omega = \omega_p) = -20 \log \frac{1}{\sqrt{1 + \varepsilon^2}}$

\[\Rightarrow \varepsilon = \sqrt{10^{\frac{A_{\text{max}}}{10}} - 1}\]

- At $\omega = \omega_s$

\[A(\omega = \omega_s) = 10 \log \left[1 + \varepsilon^2 \cosh^2 \left(N \cosh^{-1} \left(\frac{\omega_s}{\omega_p}\right)\right)\right]\]
Chebyshev LP Filter

- Transfer function

\[ T(s) = \frac{K \omega_p^N}{\epsilon 2^{N-1} (s - p_1)(s - p_2) \cdots (s - p_N)} \]

where

\[ p_k = -\omega_p \sin\left(\frac{2k - 1}{N} \frac{\pi}{2}\right) \sinh\left(\frac{1}{N} \sinh^{-1}\frac{1}{\epsilon}\right) + j\omega_p \cos\left(\frac{2k - 1}{N} \frac{\pi}{2}\right) \cosh\left(\frac{1}{N} \sinh^{-1}\frac{1}{\epsilon}\right) \]
Implementation of Higher Order Filters

• 1\textsuperscript{st} and 2\textsuperscript{nd} order filters may be cascaded to realize higher order filters

• Factor higher order transfer function into product of 1\textsuperscript{st} and 2\textsuperscript{nd} order functions

• Implement each 1\textsuperscript{st} and 2\textsuperscript{nd} order stage as an opamp circuit
### 1<sup>st</sup> Order Filter Implementation

| Filter Type and \( T(s) \) | \( s \)-Plane Singularities | Bode Plot for \(|T|\) | Passive Realization | Op Amp–RC Realization |
|-----------------------------|-----------------------------|----------------------|---------------------|----------------------|
| (a) Low pass (LP) \( T(s) = \frac{a_0}{s + \omega_0} \) | \( j\omega \) \( \sigma \) \( 0 \) \( \omega_0 \) | 20 \( \log |a_0|/\omega_0 \) \( 20 \text{ dB} \) \( -20 \text{ dB} \) \( \frac{1}{\text{decade}} \) | DC gain = 1 \( \omega_0 = \frac{1}{RC} \) | DC gain = \(-R_2 / R_1\) \( \omega_0 = \frac{1}{R_2C} \) |
| (b) High pass (HP) \( T(s) = \frac{a_1s}{s + \omega_0} \) | \( j\omega \) \( \sigma \) \( 0 \) \( \omega_0 \) | 20 \( \log a_1 \) \( 20 \text{ dB} \) \( +20 \text{ dB} \) \( \frac{1}{\text{decade}} \) | High frequency gain = 1 \( \omega_0 = \frac{1}{RC} \) | High frequency gain = \(-R_2 / R_1\) \( \omega_0 = \frac{1}{R_1C} \) |
| (c) General \( T(s) = \frac{a_1s + a_0}{s + \omega_0} \) | \( j\omega \) \( \sigma \) \( 0 \) \( \omega_0 \) | 20 \( \log \left| \frac{a_0}{a_1} \right| \omega_0 \) \( 20 \text{ dB} \) \( -20 \text{ dB} \) \( \frac{1}{\text{decade}} \) | | |

Ref. Sedra and Smith, Figure 12.13
### 2nd Order Filter Transfer Functions

| Filter Type and $T(s)$ | s-Plane Singularities | $|T|$ |
|------------------------|-----------------------|------|
| **(a) Low pass (LP)**  | ![Diagram](image) | ![Diagram](image) |
| $T(s) = \frac{a_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ | $\frac{|a_0|Q}{\sqrt{1 - \frac{1}{4Q^2}}}$ | $\omega_{max} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$ |
| DC gain = $a_0 / \omega_0^2$ | $\omega_{max}$ | $\omega_{max}$ |

| **(b) High pass (HP)** | ![Diagram](image) | ![Diagram](image) |
| $T(s) = \frac{a_2 s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ | $\frac{|a_2|Q}{\sqrt{1 - \frac{1}{4Q^2}}}$ | $\omega_{max} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$ |
| High freq. gain = $a_2$ | $\omega_{max}$ | $\omega_{max}$ |

| **(c) Bandpass (BP)** | ![Diagram](image) | ![Diagram](image) |
| $T(s) = \frac{a_1 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ | $\frac{|a_1 Q|}{\omega_0 \sqrt{1 + \frac{1}{4Q^2}}}$ | $0.707 T_{max} = \frac{(a_1 Q \sqrt{2} \omega_0)}{\omega_0}$ |
| Center-frequency gain = $\frac{a_1 Q}{\omega_0}$ | $\omega_{max} = \sqrt{1 + \frac{1}{4Q^2}}$ | $\omega_{max}$ |

Ref. Sedra and Smith, Figure 12.16
2nd Order Filter LCR Circuits

LP LCR Circuit:

\[
T(s) = \frac{1}{s^2 + s \frac{1}{RC} + \frac{1}{LC}} = \frac{a_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}
\]

HP LCR Circuit:

\[
T(s) = \frac{s^2}{s^2 + s \frac{1}{RC} + \frac{1}{LC}} = \frac{a_2 s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}
\]

- Issue: Inductors expensive and large
Opamp Simulated Inductor

\[ Z_{in} = \frac{V_1}{I_1} = s\left(\frac{C_4 R_1 R_3 R_5}{R_2}\right) = sL \]
Feedback Implementation of 2\textsuperscript{nd} Order Filter

HP Transfer Function:

\[ T(s) = \frac{V_{HP}}{V_i} = \frac{a_2 s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} = \frac{a_2}{1 + \frac{1}{Q} \left( \frac{\omega_0}{s} \right) + \left( \frac{\omega_0}{s} \right)^2} \]

\[ V_{HP} \times \left( 1 + \frac{1}{Q} \left( \frac{\omega_0}{s} \right) + \left( \frac{\omega_0}{s} \right)^2 \right) = a_2 V_i \]

\[ V_{HP} = a_2 V_i + \frac{1}{Q} \left( -\frac{\omega_0}{s} \right) V_{HP} + \left( -\frac{\omega_0}{s} \right) \left( -\frac{\omega_0}{s} \right) V_{HP} \]

Gain

Inverting integrator

Inverting integrator

Ref. Sedra and Smith, Figure 12.13
Two Integrator Feedback Loop Circuit

- Example: Kerwin-Huelsman-Newcomb two-integrator circuit

Ref. Sedra and Smith, Figure 12.24
Single Opamp 2\textsuperscript{nd} Order Filter Block

- Alternate approach
  - Single opamp with RC network in feedback path

- Sallen-Key Filter Circuit

\[
T(s) = \frac{1}{C_3 C_4 R_1 R_2} \left( s^2 + s \frac{1}{C \left( \frac{1}{R_1} + \frac{1}{R_2} \right)} + \frac{1}{C_3 C_4 R_1 R_2} \right)
\]

\[
T(s) = \frac{a_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}
\]
III. FilterPro™ Software Tool

• Purpose
  – “Although low-pass filters are vital in modern electronics, their design and verification can be tedious and time consuming. The FilterPro program is designed to aid in the design of low-pass filters implemented with the multiple feedback (MFB) and Sallen-Key topology.”

• Download link:
  http://focus.ti.com/docs/toolsw/folders/print/filterpro.html#description
Step 1: Select Filter Type

A Highpass filter allows high frequency signals to pass through and attenuates those lower than the cutoff frequency.
Step 2: Select Filter Specifications

<table>
<thead>
<tr>
<th>Filter Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain ($A_0$): 1 V/V 0 dB</td>
</tr>
<tr>
<td>Passband Frequency ($f_c$): 20 Hz</td>
</tr>
<tr>
<td>Allowable Passband Ripple ($R_p$): 0 dB</td>
</tr>
<tr>
<td>Stopband Frequency ($f_s$): 2 Hz</td>
</tr>
<tr>
<td>Stopband Attenuation ($A_{sb}$): -20 dB</td>
</tr>
</tbody>
</table>

Optional - Filter Order: Set Fixed: 2
Step 3: Filter Response

Please select a filter response:

- Gaussian to 6 dB
- Linear Phase 0.05°
- Linear Phase 0.5°
- Bessel
- Butterworth
- Gaussian to 12 dB

**Response Type** | **Order** | **No. of Stages** | **Max. Q**
---|---|---|---
Butterworth | 1 | 1 | 0.5
Gaussian to 12 dB | 1 | 1 | 0.5
Step 4: Select Filter Topology

Step 4: Filter Topology

Please select a filter topology:

- Real-Pole

Real pole topology consists of a single order pole which is real.
Component tolerances
Design Results – Bill Of Materials

Actual implementation depends on opamp specs
Simulate Design in LTSPICE

;tran 0 100ms 0 1ms
.ac dec 10 1 1000 Source

Vin

C1 820n

SINE(0 0.1 50)
AC 0.1 0

R1 8.2k

LT1635

Vout
Step 1: Select Filter Type

Please select a filter type:

- Lowpass
- Highpass
- Bandpass
- Bandstop / Notch
- Allpass (Time Delay)

A Bandstop filter attenuates frequencies around the center frequency and allows those outside the range to pass.
Step 2: Select Filter Specifications

- Gain (A_o): 10 V/V, 20 dB
- Center Frequency (f_o): 1000 Hz
- Allowable Passband Ripple (R_p): 1 dB
- Passband Bandwidth (BW_p): 1000 Hz
- Stopband Bandwidth (BW_s): 100 Hz
- Stopband Attenuation (A_s): -10 dB

Optional - Filter Order: Set Fixed 2
Step 3: Filter Response

Please select a filter response:

- Gaussian to 6 dB
- Gaussian to 12 dB
- Linear Phase 0.5°
- Linear Phase 0.05°
- Butterworth
- Bessel
- Chebychev 1 dB
- Chebychev 0.5 dB

Response Type | Order | No. of Stages | Max. Q
--- | --- | --- | ---
Gaussian to 6 dB | 2 | 1 | 1
Gaussian to 12 dB | 2 | 1 | 1
Linear Phase 0.5° | 2 | 1 | 1
Linear Phase 0.05° | 2 | 1 | 1
Step 4: Select Filter Topology

Please select a filter topology:

- Multiple-Feedback
- Sallen-Key

Sallen-Key topology is a second-order filter topology having non-inverting gain. It is commonly used in voltage-controlled voltage-source (VCVS) implementations. The gain is configurable with isolated gain resistors making this topology highly usable.
Design Results – Main Page

Component tolerances
Design Results – Bill Of Materials

<table>
<thead>
<tr>
<th>Name</th>
<th>Quantity</th>
<th>Part Number</th>
<th>Value</th>
<th>Description</th>
<th>Tolerance</th>
<th>Manufacturer</th>
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</thead>
<tbody>
<tr>
<td>R1 (Stage 1)</td>
<td>1</td>
<td>Standard</td>
<td>15KΩ</td>
<td>Resistor</td>
<td>E12: 10%</td>
<td></td>
</tr>
<tr>
<td>R2 (Stage 1)</td>
<td>1</td>
<td>Standard</td>
<td>15KΩ</td>
<td>Resistor</td>
<td>E12: 10%</td>
<td></td>
</tr>
<tr>
<td>R3 (Stage 1)</td>
<td>1</td>
<td>Standard</td>
<td>8.2KΩ</td>
<td>Resistor</td>
<td>E12: 10%</td>
<td></td>
</tr>
<tr>
<td>R4 (Stage 1)</td>
<td>1</td>
<td>Standard</td>
<td>2.2KΩ</td>
<td>Resistor</td>
<td>E12: 10%</td>
<td></td>
</tr>
<tr>
<td>R5 (Stage 1)</td>
<td>1</td>
<td>Standard</td>
<td>1.2KΩ</td>
<td>Resistor</td>
<td>E12: 10%</td>
<td></td>
</tr>
<tr>
<td>C1 (Stage 1)</td>
<td>1</td>
<td>Standard</td>
<td>10nF</td>
<td>Capacitor</td>
<td>E12: 10%</td>
<td></td>
</tr>
<tr>
<td>C2 (Stage 1)</td>
<td>1</td>
<td>Standard</td>
<td>10nF</td>
<td>Capacitor</td>
<td>E12: 10%</td>
<td></td>
</tr>
<tr>
<td>C3 (Stage 1)</td>
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<td>Standard</td>
<td>20nF</td>
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<td>OpAmp (Stage 1)</td>
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<td>Standard</td>
<td>Ideal OpAmp</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Actual implementation depends on opamp specs
Simulate Design in LTSPICE

 tran 0 100ms 0 1ms
 ac dec 100 10 10000

 SINE(0 0.1 50)
Real Opamp Requirements

Op Amp Selection

It is important to choose an op amp that can provide the necessary dc precision, noise, distortion, and speed. Texas Instruments offers an excellent selection of op amps that can be used for high performance active filters. The following internet web page provides guides to select an appropriate op amp for your application:

http://focus.ti.com/docs/analog/analoghomepage.jhtml

This same page also contains a link to the download page for FilterPro.

The following paragraphs define parameters that should be evaluated when selecting op amps for filter circuits.

Ref. Application note:

http://focus.ti.com/lit/an/sbfa001a/sbfa001a.pdf
Real Opamp Requirements

Op Amp Gain Bandwidth Product (GBP)

In a low-pass filter section, maximum gain peaking is very nearly equal to $Q$ at $f_n$ (the section’s natural frequency). So, as a rule of thumb:

For an MFB section: Op amp GBP should be at least $100 \cdot \text{GAIN} \cdot f_n$.

High-Q Sallen-Key sections require higher op amp GBP.

For a Sallen-Key section: For $Q > 1$, op amp GBP should be at least $100 \cdot \text{GAIN} \cdot Q^3 \cdot f_n$. For $Q \geq 1$, op amp GBP should be at least $100 \cdot \text{GAIN} \cdot f_n$.

For a real-pole section: Op amp GBP should be at least $50 \cdot f_n$.

Ref. Application note:

http://focus.ti.com/lit/an/sbfa001a/sbfa001a.pdf
Summary

• Assignment 3 (Active Filter)
• Filter fundamentals
• FilterPro™ Software Tool
• See detailed schedule in syllabus for assignment check-off process
• Also check for lab/TA hours at NEB 246 and E-learning